

$m^{\circ} \dots) \kappa(n+1)$

$$\dots \frac{F}{F+nd} \cdot q^{\circ} \cdot \kappa(n+1)$$

+

6., wenn man

$\mu' = \pi'$ u. f. w. setzt.

$nc) q^{\circ} \kappa(n+1)$

$(n+1)$

$(n+1)$

$(n+1)$

&c

$) \kappa(n+1)$

Eben so darf man p =
setzen.

Solglich wird P.

$$= \frac{g \cdot q^{g-1} + mf + \dots}{\dots}$$

$$= \frac{g}{G} \frac{d(q^G)}{dx}, \text{ wenn}$$

= G gesetzt wird.

Reihe für $q^g, q^G =$

$$q^G \kappa 1. x^{\frac{sG}{g}} + q^G \kappa$$

$$\text{folglich } \frac{d(q^G)}{dx} =$$

$$\cdot q^G \kappa 2. x^{\frac{s}{g}}$$

$$\text{und } \frac{d(q^G)}{dx} \kappa (n+1)$$

Carl Friedrich Hindenburg and the Combinatorial School

Hindenburg's combinatorial school, its Leibnizian dream of a finite analysis, and its afterlife in Novalis.

It is the greatest remedy for the mind if a few thoughts can be found from which infinite others arise in order, just as from the assumption of a few numbers, from one to ten, all the other numbers can be derived in order.

— Gottfried Wilhelm Leibniz, ‘Of an Organum or Ars Magna of Thinking’, in *Philosophical Writings*, ed. G. H. R. Parkinson, tr. Mary Morris and G. H. R. Parkinson (London: J. M. Dent & Sons, 1973), 1

At the twilight of the 18th century, European mathematics was still recovering from the monumental revelations of the 17th century in its discovery of the calculus. The veritable moral panic provoked by the success of infinite series methods showed no signs of abating. Before Cauchy would go on to provide a rigorous formalisation of the calculus by grounding it in the concept of the limit, the question of the foundations of analysis was up in the air. In this state of epistemological suspension a variety of different directions were pursued. One of these directions, following on from the mathematical work of Euler in his *Introductio in analysin infinitorum* of 1748, and championing Gottfried Wilhelm Leibniz’s philosophical ideal of a *characteristica universalis*, was that of Carl Friedrich Hindenburg and his combinatorial school.

From Euler’s work stemmed the notion of treating functions as objects to be manipulated algebraically with the aid of infinite series and continued fractions. Lagrange, in his *Théorie des fonctions analytiques* (1797), had based the differential and integral calculus on a power series approach. The combinatorialists took a novel approach in algebraic analysis that was called the *Analysis des Endlichen*, or ‘finite analysis’, seeking a uniquely algebraic basis for the infinitesimal calculus. They appropriated Euler’s use of the infinite series and attempted to use this as the basis upon which to construct the entire mathematical edifice. The motivation was to develop a general science of the structure of formulae utilising combinatorial analysis. There was a focus on developing new forms of notation, some of which were particularly unwieldy and indeed ugly,¹ and a pioneering employment of tabular methods for the representation of combinatorial aggregates. The school were the first to set up principles for working with tables of permutations, combinations, and variations in order to prevent the introduction of errors.

¹Florian Cajori, *A History of Mathematical Notations* (New York: Dover, 1993), 443–445.

Of prime importance for this project was the polynomial (or multinomial) formula, for which a few proofs had recently been attempted. Hindenburg gave a proof of the general polynomial formula,² or, as it was called at the time, *die Potenzierung des Infinitinoms*. Unlike previous attempts, Hindenburg's formulation had the merit of allowing for a generalisation to fractional and negative exponents.

It was an extremely important result for working with power series, because division, exponentiation, and the extraction of roots could be reworked into this form by taking the respective negative, natural, and fractional powers. Because of this, Klügel, one of Hindenburg's followers, described the polynomial formula as the 'pinnacle from which one can survey the region of analysis'.

Another member of the school, Heinrich August Rothe, showed that the solution of arbitrary or transcendental equations could be found by applying the polynomial formula to obtain what is called *series reversion*. Lagrange had already displayed the principles of this technique in what is now called the Lagrange inversion theorem, but he was more concerned with its application to astronomical phenomena. Rothe provided the more general solution. This filled the rest of the school with great excitement, because it demonstrated the efficacy of the polynomial formula in general algebraic solutions, supplying much-needed proof of work for their methodical engine.



In Leibniz, to whom multiple references are made in Hindenburg's work,³ the school found its conceptual drive. Inheriting the spirit of the *ars combinatoria*, or 'art of combining', from Ramon Llull and, later, Athanasius Kircher, Leibniz dreamt of a symbolic system that would produce complex conceptual aggregates from the rule-governed combination of simpler ones. The *ars combinatoria* would then become an *ars inveniendi*, an art of invention or discovery. In his *Dissertatio de arte combinatoria* (1666), written at Leipzig University, where Hindenburg would later become professor of physics, Leibniz outlines a schematic for such a system. Though an early work, more prospective than methodological, its theme would haunt his later writings, insinuating itself constantly as a kind of *peradam* or philosophers'

²Carl Friedrich Hindenburg, *Infinitinonii dignitatum exponentis indeterminati historia, leges ac formulae* (Göttingen, 1779).

³Carl Friedrich Hindenburg, ed., *Sammlung combinatorisch-analytischer Abhandlungen* (Leipzig, 1796–1800).

stone. Later, Gödel would go so far as to suggest that this pursuit fed directly into Leibniz's work on the calculus. In 'Of Universal Synthesis and Analysis', Leibniz writes:

The art of combinations in particular is, in my opinion, the science which treats of the forms of things or of formulae in general (it could also be called generally the science of symbols, or of forms. That is, it is the science of *quality* in general, or, of the like and the unlike [...]) It is distinguished from algebra, which is concerned with formulae applied to *quantity*, i.e. with the equal and the unequal. Consequently, algebra is subordinate to the art of the combinations, and constantly uses its rules.⁴

It is the final statement here that is key. The school saw combinatorics as a global method that would subsume algebra as a local subdiscipline. For them, combinatorial theory could serve as the basis for pure mathematics and tackle its critical questions. This ambition for combinatorics naturally seems somewhat absurd today. Philippe Séguin links it to the post-Kantian spirit of the age.⁵ Since we are talking about the climate in which Fichte had sought to root the science of consciousness in the axiom $A = A$, it seems only natural that Hindenburg, not himself a considerable mathematician, would seek to ground all of algebra on the polynomial formula. There was, apparently, a tendency towards the universalisation of quite particular results. Either way, the general mood was to rid analysis of geometry and so-called "intuition", as a reaction against Kant's attempt to ground mathematics in the formal intuitions of time and space. The return to the rationalist Leibniz makes sense in this light. Adopting a different perspective, if we consider the manner in which mathematics would continue to develop with the introduction of the limit concept and the expansion of the role of imaginary numbers, the school may even have been prescient in its move towards symbolic abstraction and away from geometric intuition.

As a method of mathematical education, the ways of the combinatorial

⁴Gottfried Wilhelm Leibniz, 'Of Universal Synthesis and Analysis', in *Philosophical Writings*, ed. G. H. R. Parkinson, tr. Mary Morris and G. H. R. Parkinson (London: J. M. Dent & Sons, 1973).

⁵Philippe Séguin, 'La recherche d'un fondement absolu des mathématiques par l'École combinatoire de C. F. Hindenburg (1741–1808)', *Philosophia Scientiae*, cahier spécial 5 (2005).

school lived on in Germany well into the 19th century.⁶ In some ways, despite the labyrinthine complexity of the notation in Hindenburg's proofs, the focus on series was a good means of illustrating the application of elementary mathematical methods at varying degrees of abstraction. As Jahnke has it, it formed a part of the *Denkwelt* (cognitive universe) of German mathematical *Bildung* during the period, long after the school fell out of favour.



Although they yielded some concrete results, as described above, and must be included in any exhaustive history of the development of modern mathematics, the combinatorial school are a strong candidate for the most reviled movement in the standard reference works. In a subchapter entitled 'Ridiculous Interlude' (!), E. T. Bell writes that

The combinatorial school headed by C. F. Hindenburg (1741–1808), was the unlovely offspring of two human failings, neither of which is popularly supposed to have any relevance for the sublimities of pure mathematics: blind hero worship and national jealousy.⁷

While Bell is correct in locating the ultimate failure of the school in its dogmatic philosophical foundations, he is of course writing shortly before the re-emergence of combinatorics as a dignified discipline in its own right, thanks to the age of computation and the subsequent vindication of discrete mathematics. As for "blind hero worship", there seems to be little documentary evidence to support any character assertions about Hindenburg besides his prodigious capacity as an organiser of scholars and journals. There is equally something fishy about the attribution of national jealousy here. It is certain that German mathematics suffered a drought of international superstars in the 18th century, and it would not be until the ascent of Gauss at the dawn of the 19th century that the country would find itself on an equal footing with France and England again. However, this trend towards sensationalising personality cults and national rivalries, a hallmark of the Newton–Leibniz literature, is always extremely speculative and never particularly useful.

Elsewhere, Cajori, in his characterisation of the period's experimentation

⁶Hans Niels Jahnke, 'Algebraic Analysis in Germany, 1780–1840: Some Mathematical and Philosophical Issues', *Historia Mathematica* 20 (1993), 265–284.

⁷E. T. Bell, *The Development of Mathematics* (London: McGraw-Hill, 1945), 290.

with infinite series, notes that

Euler reached some very pretty results on infinite series, now well known, and also some very absurd results, now quite forgotten. The faults of his time found their culmination in the Combinatorial School in Germany, which has now passed into deserved oblivion.⁸

This seems like far too summary a dismissal, since Hindenburg, Rothe, and Kramp's results are still used today. In *The Art of Computer Programming*, volume 4A, Donald Knuth writes of 'Hindenburg's hype', insisting that his 'small successes led him to believe that he had made revolutionary advances in mathematics—although he did condescend to remark that other people such as de Moivre, Euler, and Lambert had come close to making similar discoveries'.⁹ There is something suspicious about this disavowal, which seems like a cribbing of the previous histories, because elsewhere in the tome Knuth cites no fewer than three of Hindenburg's works and one of Rothe's. A vindication of certain of the school's methods in the light of the significance of combinatorial methods for computer science is perhaps overdue. This roster of comments is worth clipping and comparing, for there is an implacably widespread hostility shown towards Hindenburg and his followers amongst historians of mathematics.



Hindenburg's influence fared rather better in philosophy. Given his fervid admiration for Leibniz, this is perhaps fitting. For example, in his *Schlußanmerkung*, Salomon Maimon addresses with some interest the controversy that had arisen between Hindenburg and Wenceslaus Johann Gustav Karsten over the former's treatment of Euclidean parallels.¹⁰ It is equally certain that the bizarre mathematical mystic Józef Maria Hoene-Wroński, another target of the rather argumentative Bell, was familiar with the methods of the combinatorial school. However, it is in the work of Novalis, who almost certainly studied under Hindenburg at Leipzig, that his presence is felt most keenly. We

⁸ Florian Cajori, *A History of Mathematics* (London: Macmillan & Co., 1909), 390.

⁹ Donald Knuth, *The Art of Computer Programming*, vol. 4A: *Combinatorial Algorithms, Part 1* (Boston: Addison-Wesley, 2011), 507–508.

¹⁰ Gideon Freudenthal, *Definition and Construction: Salomon Maimon's Philosophy of Geometry* (Berlin: Max Planck Institute for the History of Science, 2006), 106.

know for certain that Novalis owned Hindenburg's books, as well as those of other members of the school. In *Das allgemeine Brouillon*, his project for a Romantic encyclopaedia, Novalis cites Hindenburg a number of times and posits combinatorial analysis as a model of conceptual synthesis in general. In fragment 547, entitled 'Musical Mathematics', we find the following:

Combinatorial analysis leads to numerical imaginings—and teaches the art of the composition of numbers—mathematical basso continuo. (Pythagoras. Leibniz). Language is a musical instrument of ideas. The poet, rhetorician and philosopher play and compose grammatically. A fugue is thoroughly logical or scientific—It may also be treated poetically.

Basso continuo includes musical algebra and analysis. Combinatorial analysis is critical algebra and analysis—and the theory of musical composition stands in the same relation to basso continuo, as combinatorial analysis does to simple analysis.

Many mathematical problems cannot be solved in isolation, but only in combination with other problems—from a higher point of view—simply through a combinatorial operation.¹¹

It is this project for a Romantic encyclopaedia that captures the philosophical direction of the combinatorial school most vividly. In fact, Novalis's formula for Romanticisation was that of raising the object of enquiry to a higher power, *Potenzierung*: a clear nod to the school's methods. In Novalis's dream of a coalition between the arts and sciences we see something of the school's extreme ambitions regarding the foundations of mathematics. In what is a clear restatement of Hindenburg's intent with the polynomial formula, Novalis remarks that 'science on the whole is generally the total function of the data and the facts—the n -th power of the binomial series of the data and the facts'.¹² At the suggestion that all the world's knowledge could be contained within a power series expansion, we truly find ourselves in the outer reaches of philosophical speculation.

The proto-foundationalist project of the combinatorial school was a failure. In this, their dogmatic Leibnizianism was a dead end. However, there was truly something of the spirit of *composition* found in contemporary pure

¹¹Novalis, *Notes for a Romantic Encyclopaedia: Das Allgemeine Brouillon*, ed. and tr. David W. Wood (Albany: State University of New York Press, 2007), 96–97.

¹²Novalis, *Notes for a Romantic Encyclopaedia*, 30.

mathematics in their grasping for the infinite via the combination and modulation of discrete elements, in their attempts to raise so-called finite analysis to a series of higher powers. It is this spirit that is undeserving of the layers of snark and disdain that Hindenburg and his peers have accrued with the passage of time. It is here that methodical conceptual reanimation can provide some due dignity.



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